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De pressione ponderis in planum cui incumbit

Leonhard Euler

DE PRESSIONE PONDERIS IN PLANVM CVI INCVMBIT.

Auctore

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1.

Quantam preſſionem planum a pondere incumbente ſuſtineat, in elementis doceri ſolet, ſcilicet ſi planum fuerit horizontale preſſionem ipſi ponderi eſſe aequalem, ſin autem ad horizontem ſit inclinatum, eam preſſionem in ratione ſinus totius ad coſinum inclinationis eſſe minuendam; tum vero vtroque caſu directionem preſſionis in planum eſſe normalem, et per centrum grauitatis corporis tranſire. Hoc autem de tota tantum preſſione, quam planum ſuſtinet, eſt intelligendum; neutiquam vero ab Auctoribus definitur, quantis viribus ſingula plani puncta, quibus pondus ſuſtinetur, vrgeantur.

2. Haud equidem memini ſimpliciſſimum caſum, quo pondus ternis pedibus plano inſiſtit, euolutum videre; quem autem ſequenti modo ſatis concinne expedire licet: Inſiſtant plano terni pedes in punctis A, B, C et recta ex centro grauitatis ad planum normaliter ducta cadat in punctum O, tum Tab. II. Fig. 1.

ductis rectis O A, O B, O C, item lateribus A B, B C, C A; tota preſſio ſe habebit ad preſſionem in puncto A, vel B, vel C, quemadmodum area totius trianguli A B C ad aream trianguli, ſiue B O C, ſiue A O C, ſiue A O B; ex quo intelligitur preſſiones ſingulorum pedum inter ſe aequales non fore, niſi punctum O in ipſum centrum grauitatis trianguli A B C incidat.

3. Verum ſi pondus quatuor pedibus plano inſiſtat, determinatio ſingularum preſſionum non ſolum multo magis ardua deprehenditur, ſed etiam prorſus incerta et lubrica videtur; ſtatim enim ac illi pedes, non exactiſſime inter ſe fuerint aequales, ita vt omnes plano pariter innitantur, manifeſtum eſt totum pondus a ternis tantum pedibus ſuſtentari et quartum penitus fore ſuperfluum; atque haec incertitudo multo magis locum habet, ſi numerus pedum adhuc fuerit maior, vel ſi pondus baſi quaedam continua plano incumbat, tum enim niſi tam ipſum planum, quam baſis corporis perfecte inter ſe congruant et leuiſſimae aſperitates in iis promineant plerumque totum pondus in tribus tantum punctis ſuſtentabitur.

4. Ne autem perfectiſſima illa pedum aequalitas, qualem vix admittere licet, negotium faceſſat, concipiamus planum ſiue ſolum cui pondus incumbit, non adeo eſſe durum, vt nullam plane impreſſionem recipere poſſit, ſed quaſi panno eſſe obductum, cui pedes illi aliquantillum ſe immergere queant;

Principium Generale.

Tab. II. Fig. 3. 6. Siue pondus pluribus pedibus innitatur, siue basi incumbat plana cuiuscunque figurae, sit punctum M siue extremitas cuiuspiam pedis, siue elementum quodpiam basis pro quo pressio quaeritur. Concipiatur ibi perpendiculariter erecta linea $M\mu$ ipsi pressioni proportionalis, atque necesse est omnia ista puncta μ in quopiam plano terminari; hoc igitur principio stabilito, quemadmodum pro omnibus casibus pressionem in singulis basis punctis definiri oporteat, hic sum expositurus.

7. Primum igitur in indolem plurium atque adeo infinitorum punctorum in eodem plano existentium inquiramus, quem in finem sit recta FG intersectio, qua planum cui pondus incumbit, a plano illo per omnia puncta μ transeunte intersecatur, quae quasi vti incognita spectari debet; sumamus pro lubitu axem quendam fixum AB, ad quem positionem punctorum M referamus ope normalium MN ad hunc axem ductarum, ac vocemus coordinatas $AN = x$, $NM = y$ et ipsum perpendiculum $M\mu$ pressionem referens $= z$, ita vt punctum μ more solito ternis coordinatis x, y et z inter se normalibus definiatur. Tum vero pro intersectione ante memorata FG, ponamus spatium $AF = f$, angulum $AFG = \zeta$, inclinationem autem binorum planorum $= \theta$. Iam ex puncto M, pariterque ex N ad hanc rectam FG ducantur normales MV, NL, et NT parallela ipsi FG, ac iungatur recta μV. Nunc igitur

igitur ex triangulo N F L, obtinemus ob

$$TN = f + x,\ FL = (f+x)\,\text{cof.}\,\zeta,\ NL = (f+x)\,\text{fin.}\,\zeta;$$

tum vero ex triangulo M N T vbi angulus N M T itidem est ζ, colligimus

$$NT = y\,\text{fin.}\,\zeta,\ MT = y\,\text{cof.}\,\zeta,$$

hinc itaque concludimus

$$MV = y\,\text{cof.}\,\zeta + (f+x)\,\text{fin.}\,\zeta \text{ et}$$

$$FV = (f+x)\,\text{cof.}\,\zeta - y\,\text{fin.}\,\zeta;$$

quare quum sit angulus $MV\mu = \theta$, consequimur

$$z = (f+x)\,\text{fin.}\,\zeta.\,\text{Tang.}\,\theta + y\,\text{cof.}\,\zeta.\,\text{Tang.}\,\theta.$$

8. Hinc igitur intelligitur relationem ternarum coordinatarum x, y et z semper huiusmodi aequatione expressum iri $z = \alpha + \beta x + \gamma y$, vbi scilicet litterae α, β, γ sunt constantes; comparatione autem huius formulae cum ante inuenta, instituta, adipiscimur hos valores

$$\alpha = f\,\text{fin.}\,\zeta.\,\text{Tang.}\,\theta;\ \beta = \text{fin.}\,\zeta\,\text{Tang.}\,\theta;\ \gamma = \text{cof.}\,\zeta\,\text{Tang.}\,\theta,$$

vnde vicissim ex cognitis α, β, et γ innotescunt valores

$$f = \frac{\alpha}{\beta};\ \text{Tang.}\,\zeta = \frac{\beta}{\gamma};\ \text{Tang.}\,\theta = \frac{\beta}{\text{fin.}\,\zeta} = \frac{\gamma}{\text{cof.}\,\zeta},$$

quibus positio plani per puncta μ transeuntis determinatur.

Problema Generale.

9. Quaecunque fuerit figura basis $fgbk$, qua corpus quodpiam solo plano incumbit, inuestigare omnes pressiones, quas singula basis elementa, sustinent. Tab. II. Fig. 4.

O o 3 Solutio.

Solutio.

Denotet G pressionem totalem corporis incumbentis, et recta ex eius centro grauitatis in planum perpendiculariter demissa, incidat in punctum O; ac sumtis pro arbitrio binis axibus AB ac AC inter se normalibus, ad eos ex O agantur perpendiculares OF et OG, vocenturque $AF = f$ et $AG = g$; tum vero pro puncto basis quocunque M ponantur coordinatae $AX = x$ et $XM = AY = y$, ipsa autem pressio quaesita in puncto M vocetur $= z$; modo autem vidimus, poni oportere $z = \alpha + \beta x + \gamma y$. Quum autem haec pressio z respondeat elemento basis Mm cuius areola $= dx\,dy$, ipsa pressio quam haec areola sustinet, erit $z\,dx\,dy$; cuius integrale ob geminam variabilem x et y bis sumtum, pressioni totali hoc est ponderi G aequale statui debet; hoc autem integrale duplicatum more recepto repraesentemus per $\iint z\,dx\,dy$, ita vt esse debeat $\iint z\,dx\,dy = G$, ideoque loco z eius valore substituto habebimus hanc aequationem:

$$\alpha \iint dx\,dy + \beta \iint x\,dx\,dy + \gamma \iint y\,dx\,dy = G.$$

10. Hac aequatione autem effectus pressionis nondum exhauritur, sed insuper necesse est, vt etiam summa omnium momentorum Elementarium respectu cuiusuis axis, aequetur momento pressionis totalis G in puncto O applicatae, respectu eiusdem axis; sufficit autem hanc aequalitatem pro binis tantum axibus AB et AC constituisse, quandoquidem demonstratum est, eam ad omnes axes vtcunque assum-

assumtos exten
primo ad axen
nis totalis fit =
$= z\,y\,dx\,dy$,
sumendis, esse
$\alpha \iint y\,dx\,dy +$
Simili modo re
nis totalis est
$= x\,z\,dx\,dy$, it
siue euoluendo:
$\alpha \iint x\,dx\,dy +$

11. Quod
totam basin fg
tendantur, tres
I. $\alpha \iint dx\,d$
II. $\alpha \iint y\,dx\,d$
III. $\alpha \iint x\,dx\,d$
ex quibus ternas
dite definire licet
que baseos punc
sustinet, erit z
pressio in singulis

12. Talibus
tum tantum opu
per spatium aliqu
dem casu saepenu

assumtos extendi. Referamus ergo haec momenta primo ad axem A B, pro quo momentum pressionis totalis fit $= Gg$, pressionis autem elementaris $= zy\,dx\,dy$, ita vt integralibus duplicatis, vt ante sumendis, esse debeat $\int\int yz\,dx\,dy = Gg$, siue

$$\alpha\int\int y\,dx\,dy + \beta\int\int xy\,dx\,dy + \gamma\int\int yy\,dx\,dy = Gg.$$

Simili modo respectu axis A C, momentum pressionis totalis est Gf, pressionis vero elementaris $= xz\,dx\,dy$, ita vt esse debeat $\int\int zx\,dx\,dy = Gf$, siue euoluendo:

$$\alpha\int\int x\,dx\,dy + \beta\int\int xx\,dx\,dy + \gamma\int\int xy\,dx\,dy = Gf.$$

11. Quod si ergo singula haec integralia per totam basin $fghk$, cuiuscunque fuerit figurae, extendantur, tres resultabunt aequationes:

$$\text{I.}\quad \alpha\int\int dx\,dy + \beta\int\int x\,dx\,dy + \gamma\int\int y\,dx\,dy = G$$

$$\text{II.}\quad \alpha\int\int y\,dx\,dy + \beta\int\int xy\,dx\,dy + \gamma\int\int yy\,dx\,dy = Gg$$

$$\text{III.}\quad \alpha\int\int x\,dx\,dy + \beta\int\int xx\,dx\,dy + \gamma\int\int xy\,dx\,dy = Gf$$

ex quibus ternas nostras incognitas α, β, γ expedite definire licebit, quibus inuentis, pro quocunque baseos puncto M, pressio quam planum ibi sustinet, erit $z = \alpha + \beta x + \gamma y$, hocque modo pressio in singulis baseos punctis innotescet.

Scholion.

12. Talibus integrationibus autem duplicatis, tum tantum opus est, quando corpus basin habet, per spatium aliquod continuum extensam, quo quidem casu saepenumero euenire potest, vt calculus ob

ob figuram basis irregularem, nequidem euolui possit, quando autem corpus aliquot pedibus plano insistit, tum nulla plane integratione erit opus, cum terminos singulorum pedum, tamquam puncta spectare licet et formulae nostrae tantum ad singulos pedes seorsim sunt accommodandae, cuiusmodi quidem casus ante sumus tractaturi, quam ad basin continuae extensionis progrediamur. Neque vero absolute opus est, vt bini illi axes A B et A C ad quos momenta retulimus, sint inter se normales, sed iis quoque obliquitatem quamcunque tribuere licet; dum modo etiam coordinatae x et y eandem obliquitatem inter se seruent. Quia enim tum vniuersus calculus ad praecedentem casum reduceretur singulas distantias oblique sumtas per sinum obliquitatis multiplicando; euidens est aequationes nostras tum per eundem sinum diuisibiles fore, ita vt totus calculus nullam inde mutationem sit subiturus.

Problema I.

13. Si pondus plano incumbat in tribus punctis A, B, C definire pressionem in singulis punctis.

Solutio.

Tab. II. Fig. 5. Directio pressionis totalis, quae sit $= G$, cadat in punctum O ex quo binis lateribus A B et A C agantur parallelae O P et O Q et secundum has directiones, constituamus nostras coordinatas x et y initio sumto in puncto A. Pro hoc ergo puncto

puncto A erit x:
$= \alpha$. Pro puncto
hincque pressio in
puncto C ob $x = o$
$+ \gamma$ A C, quamobr

$G = 3\alpha + \beta$ A

Momenta autem ref
praebent hanc secun

G. O P $= \alpha$ A C

Tertia denique aequ
A C colligitur

G. O Q $= \alpha$. A

Ex tertia colligimus

β. A B $= \frac{G.OQ}{AB}$ -

ex secunda vero

γ. A C $= \frac{G.OP}{AC}$ -

qui valores in prim

$G = \frac{G.OQ}{AB} + \frac{G.}{A}$

hincque

$\alpha = G(1 - \frac{OQ}{AB}$ -

quae est pressio in
puncto B, quae erat

$\alpha + \beta$. A B, fit

ac denique pressio in

$\alpha + \gamma$. A C nun

puncto A erit $x = 0$ et $y = 0$, ideoque pressio $= \alpha$. Pro puncto B habebimus $x = AB$ et $y = 0$, hincque pressio in hoc loco $= \alpha + \beta . AB$, at pro puncto C ob $x = 0$ et $y = AC$ pressio erit $= \alpha + \gamma AC$, quamobrem prima aequatio erit

$$G = 3\alpha + \beta AB + \gamma AC.$$

Momenta autem respectu axis AB oblique sumta, praebent hanc secundam aequationem

$$G . OP = \alpha AC + \gamma AC^2.$$

Tertia denique aequatio ex momentis respectu lateris AC colligitur

$$G . OQ = \alpha . AB + \beta . AB^2.$$

Ex tertia colligimus

$$\beta . AB = \frac{G . OQ}{AB} - \alpha,$$

ex secunda vero

$$\gamma . AC = \frac{G . OP}{AC} - \alpha,$$

qui valores in prima substituti praebent

$$G = \frac{G . OQ}{AB} + \frac{G . OP}{AC} + \alpha,$$

hincque

$$\alpha = G\left(1 - \frac{OQ}{AB} - \frac{OP}{AC}\right) = G\left(1 - \frac{AP}{AB} - \frac{AQ}{AC}\right)$$

quae est pressio in ipso puncto A, pressio autem in puncto B, quae erat

$$\alpha + \beta . AB, \text{ fit } = \frac{G . OQ}{AB} = \frac{G . AP}{AB};$$

ac denique pressio in C quae erat

$$\alpha + \gamma . AC \text{ nunc fit } = \frac{G . AQ}{AC}.$$

Coroll.

14. Haec solutio cum supra data egregie conuenit, cum enim pressio totalis G, sit ad pressionem in puncto $B :: AB : AP$, hoc est vt area trianguli ABC ad aream trianguli APC, iam vero triangulum $AOC =$ triangulo APC, erit ergo tota pressio G ad pressionem in B, vt area totius trianguli ABC ad aream trianguli AOC.

Problema 2.

15. Si pondus plano incumbat in quatuor punctis A, B, C, D secundum angulos parallelogrammi dispositis, definire pressionem in singulis his punctis.

Solutio.

Incidat vis totalis $= G$ perpendiculariter in puncto O in planum, capiantur nostrae coordinatae secundum latera parallelogrammi AB et AD quibus ex O parallelae ducantur OP et OQ, ac sumto initio in A, pro hoc puncto A ambae coordinatae x et y euanescent. Pro puncto B habebimus $x = AB$ et $y = 0$, tum pro puncto C erit $x = AB$ et $y = BC = AD$, denique pro quarto puncto D fit $x = 0$ et $y = AD$, vnde pressiones quaesitae in his quatuor punctis erunt:

pro puncto $A = \alpha$; pro puncto $B = \alpha + \beta . AB$

pro puncto $C = \alpha + \beta . AB + \gamma . AD$; pro puncto $D = \alpha + \gamma . AD$

sicque

ſicque prima aequatio ita ſe habebit:

$$G = 4\alpha + 2\beta . AB + 2\gamma . AD.$$

Iam reſpectu axis AB momentum totale eſt G.OP, preſſionum autem in A et B momenta euaneſcunt, in punctis autem C et D duci debent in AD vel CB, vnde noſtra ſecunda aequatio erit:

$$G . OP = 2\alpha AD + 2\gamma . AD^2 + \beta . AB . AD.$$

Tertia autem aequatio ex momentis reſpectu axis AD ſumtis fiet:

$$G . OQ = 2\alpha . AB + 2\beta . AB^2 + \gamma . AB . AD.$$

Quum igitur binae poſteriores coniunctae praebeant:

$$G\left(\frac{AP}{AB} + \frac{AQ}{AD}\right) = 4\alpha + 3\beta . AB + 3\gamma . AD,$$

cuius duplum a triplo primae ſubtractum relinquit:

$$G\left(3 - \frac{2AP}{AB} - \frac{2AQ}{AD}\right) = 4\alpha;$$

ſicque iam inuenimus fore preſſionem in puncto

$$A = \frac{G}{4}\left(3 - \frac{2AP}{AB} - \frac{2AQ}{AD}\right),$$

quae expreſſio facile in hanc transformatur:

$$\frac{G}{4}\left(\frac{2BP}{AB} + \frac{2DQ}{AD} - 1\right).$$

Pro reliquis punctis producamus rectas PO et QO in R et S, et omnes quatuor preſſiones quaeſitae ita commodiſſime exprimi videntur:

I. Preſſio in $A = \frac{1}{4} G\left(\frac{2BP}{BA} + \frac{2DQ}{DA} - 1\right)$

II. Preſſio in $B = \frac{1}{4} G\left(\frac{2AP}{AB} + \frac{2CS}{CB} - 1\right)$

III. Preſſio in $C = \frac{1}{4} G\left(\frac{2BS}{BC} + \frac{2DR}{DC} - 1\right)$

IV. Preſſio in $D = \frac{1}{4} G\left(\frac{2CR}{CD} + \frac{2AQ}{AD} - 1\right).$

Coroll. 1.

16. Fieri igitur poteſt, vt in vno horum quatuor punctorum preſſio euaneſcat, etiamſi punctum O non extra parallelogrammum cadat, in puncto namque A preſſio fiet nulla ſi $\frac{BP}{BA} + \frac{DQ}{DA} = \frac{1}{2}$, id quod innumerabilibus modis fieri poteſt, inter quos ſimpliciſſimus eſt, vbi $BP = \frac{1}{4} AB$ et $DQ = \frac{1}{4} DA$, hoc ſcilicet caſu punctum O ita ſitum erit in diagonali AC, vt eius diſtantia a puncto C ſit quarta pars ipſius diagonalis AC.

Coroll. 2.

Tab. II. Fig. 7. 17. Operae autem pretium eſt, omnia loca O inueſtigare, quibus preſſio in puncto A euaneſcit, ex ipſa autem aequatione $\frac{2BP}{BA} + \frac{2DQ}{DA} = 1$, patet facto $BP = 0$, vt punctum O in rectam BC incidat, et quia tum $DQ = \frac{1}{2} DA$, punctum O praeciſe in punctum medium E lateris BC incidere. Similique modo ſumto $DQ = 0$, patet punctum O in medium lateris DC quod ſit F incidere, quum igitur locus omnium punctorum O ſit ad lineam rectam, omnia haec puncta cadent in rectam EF, vnde manifeſtum eſt, quoties punctum O inciderit in rectam EF, tum preſſionem in puncto A ſemper fore nullam, atque hinc ſimul intelligitur, ſi punctum O vltra hanc rectam EF ſiue intra triangulum CEF cadat, tum preſſionem in puncto A, adeo prodire negatiuam.

Scho-

Scholion.

18. Hic caſus eo magis eſt memorabilis, quod in praxi, nullam certe preſſionem negatiuam concipere licet. Quocirca imprimis nobis erit inquirendum, quid huiusmodi caſibus ſit reuera euentum. Hunc in finem incipiamus a caſu, quo punctum O in ipſam rectam E F incidit, et quia tum preſſio in puncto A plane ſit nulla, res eodem vtique redit, ac ſi pes huic puncto inſiſtens plane abeſſet, et pondus in tribus tantum punctis B, C, D ſuſtentaretur. Hoc idem vero multo magis eueniet, ſi punctum O intra triangulum C E F vbicunque inciderit, et quia tum certi ſumus, totum pondus a tribus tantum punctis B, C, D ſuſtineri; preſſio in his punctis eodem prorſus modo ſe habebit, vti in Problemate praecedente eſt definita. Hic igitur notaſſe iuuabit, tum demum omnes quatuor pedes ad onus ſuſtentandum concurrere, ſi punctum O intra parallelogrammum inſcriptum E F G H cadat, vbicunque enim extrinſecus veluti in triangulo C E F reperiatur, pes oppoſitus pro ſuperfluo haberi debebit.

Problema 3.

Fulciatur pondus octo pedibus, quorum quatuor in angulis A, B, C, D parallelogrammi plano inſiſtant, reliqui vero E, F, G, H in puncta media inter illos cadant, ita vt latera parallelogrammi in his punctis bifariam ſecentur; definire preſſiones in ſingulis his punctis. Tab. II. Fig. 8.

 Solutio.

Solutio.

19. Ducamus rectas E G et F H se mutuo in I secantes, quas pro nostris axibus assumamus, et ex puncto O iis parallelas agamus O P et O Q, atque initio in puncto I constituto, abscissas positiuas x dextrorsum, negatiuas vero sinistrorsum, tum vero applicatas positiuas y sursum, at negatiuas deorsum capiamus. His positis pressiones in singulis punctis ita se habebunt:

I. Pressio in $A = \alpha - \beta . IH - \gamma . IE$
II. Pressio in $B = \alpha + \beta . IF - \gamma . IE$
III. Pressio in $C = \alpha + \beta . IF + \gamma . IG$
IV. Pressio in $D = \alpha - \beta . IH + \gamma . IG$
V. Pressio in $E = \alpha \qquad - \gamma . IE$
VI. Pressio in $F = \alpha + \beta . IF$
VII. Pressio in $G = \alpha \qquad + \gamma . IG$
VIII. Pressio in $H = \alpha - \beta . IH.$

Quarum pressionum omnium summa est $G = 8 . \alpha$, vnde si pressio totalis fuerit $= G$, statim habemus $\alpha = \frac{1}{8} G$, quae est nostra aequatio prima. Nunc spectemus singula momenta respectu axis F I H, ac primo coniunctim consideremus vires in A et D, quarum vtraque tribus constat partibus, ac primae quidem partes α, se mutuo in aequilibrio tenentes, eodem modo partes secundae $\beta . IH$ se mutuo destruunt, vnde momentum tantum ex tertiis partibus est aestimandum, priorem ducendo in $- IE$, posteriorem vero in $+ IG$, vnde nascitur momentum $2 \gamma . IG^2$. Eodem modo ex viribus in B et C resul-

resultabit quoque idem momentum $2\gamma . IG^2$. Deinde ex viribus E et G illam in $-EI$, hanc vero in $+IE$ ducendo, emergit momentum $+2\gamma . IG^2$. Ex postremis viribus F et H autem nullum oritur momentum. Quum ergo pressionis totalis G momentum respectu eiusdem axis sit $G.OP = G.IQ$, inde colligimus hanc aequationem secundam:

$$G.IQ = 6\gamma . IG^2 \text{, ideoque } \gamma = \frac{G.IQ}{6\,IG^2}.$$

Respectu autem alterius axis per simile ratiocinium deducimur ad hanc aequationem:

$$G.IP = 6\beta . IF^2 \text{, vnde } \beta = \frac{G.IP}{6\,IF^2}.$$

Quibus inuentis pressiones in singulis octo punctis se habebunt, vt sequuntur:

I. Pressio in $A = \frac{1}{2}G\left(\frac{1}{4} - \frac{1}{3}\frac{PI}{IF} - \frac{1}{3}\frac{IQ}{IG}\right)$

II. Pressio in $B = \frac{1}{2}G\left(\frac{1}{4} + \frac{1}{3}\frac{PI}{IF} - \frac{1}{3}\frac{IQ}{IG}\right)$

III. Pressio in $C = \frac{1}{2}G\left(\frac{1}{4} + \frac{1}{3}\frac{PI}{IF} + \frac{1}{3}\frac{IQ}{IG}\right)$

IV. Pressio in $D = \frac{1}{2}G\left(\frac{1}{4} - \frac{1}{3}\frac{PI}{IF} + \frac{1}{3}\frac{IQ}{IG}\right)$

V. Pressio in $E = \frac{1}{2}G\left(\frac{1}{4} - \frac{1}{3}\frac{IQ}{IG}\right)$

VI. Pressio in $F = \frac{1}{2}G\left(\frac{1}{4} + \frac{1}{3}\frac{PI}{IF}\right)$

VII. Pressio in $G = \frac{1}{2}G\left(\frac{1}{4} + \frac{1}{3}\frac{IQ}{IG}\right)$

VIII. Pressio in $H = \frac{1}{2}G\left(\frac{1}{4} - \frac{1}{3}\frac{PI}{IF}\right)$.

Coroll.

20. Si breuitatis gratia ponatur $IF = a$; $IG = b$; $IP = p$; $IQ = q$ istae vires succinctius ita repraesentari possunt.

I.

I. Pressio in $A = \frac{1}{24} G\left(3 - \frac{4p}{a} - \frac{4q}{b}\right) = \frac{1}{24} G\left(\frac{4(a-p)}{a} + \frac{4(b-q)}{b} - 5\right)$

II. Pressio in $B = \frac{1}{24} G\left(3 + \frac{4p}{a} - \frac{4q}{b}\right) = \frac{1}{24} G\left(\frac{4(a+p)}{a} + \frac{4(b-q)}{b} - 5\right)$

III. Pressio in $C = \frac{1}{24} G\left(3 + \frac{4p}{a} + \frac{4q}{b}\right) = \frac{1}{24} G\left(\frac{4(a+p)}{a} + \frac{4(b+q)}{b} - 5\right)$

IV. Pressio in $D = \frac{1}{24} G\left(3 - \frac{4p}{a} + \frac{4q}{b}\right) = \frac{1}{24} G\left(\frac{4(a-p)}{a} + \frac{4(b+q)}{b} - 5\right)$

V. Pressio in $E = \frac{1}{24} G\left(3 - \frac{4q}{b}\right) = \frac{1}{24} G\left(\frac{4(b-q)}{b} - 1\right)$

VI. Pressio in $F = \frac{1}{24} G\left(3 + \frac{4p}{a}\right) = \frac{1}{24} G\left(\frac{4(a+p)}{a} - 1\right)$

VII. Pressio in $G = \frac{1}{24} G\left(3 - \frac{4p}{a}\right) = \frac{1}{24} G\left(\frac{4(a-p)}{a} - 1\right)$

VIII. Pressio in $H = \frac{1}{24} G\left(3 + \frac{4q}{b}\right) = \frac{1}{24} G\left(\frac{4(b+q)}{b} - 1\right)$.

Scholion.

21. Huiusmodi casibus, quibus pondus pluribus pedibus plano insistit, fusius non immoramur; antequam autem bases per spatium aliquod planum extensas, consideremus, casus quosdam quasi intermedios examinemus, quibus pondus deorsum definit in limbum quempiam siue rectilineum, siue curuilineum, vbi quidem a rectilineis incipere conuenit, quae inuestigatio, quo minorem difficultatem, ob figuram talis basis polygonae facessat; exordiamur ab vnica linea recta per cuius singula puncta tam pressiones, quam momenta respectu binorum axium fixorum inuestigemus in sequenti Lemmate.

Lemma.

Tab. II. Fig. 9. Constitutis binis axibus A B et A C inter normalibus, quorum respectu momenta sunt aestimanda, sit recta Ff portio limbi, quo pondus plano innititur; inueſtigare preſſiones per totam hanc lineam, earumque momenta reſpectu binorum axium AB et AC.

Solutio.

22. Pro puncto huius rectae quocunque Y, vocemus noſtras coordinatas $AX = x$ et $XY = y$ et per principium ſupra ſtabilitum preſſio in hoc puncto erit $\alpha + \beta x + \gamma y$, iam quum haec linea Ff ſit recta, inter has coordinatas dabitur huiusmodi aequatio $y = e + nx$, ita vt ſit $dy = n\,dx$. Quia nunc praefata preſſio per elementum Yy extenditur, ob

$$Yy = \sqrt{(dx^2 + dy^2)} = dx\sqrt{(1 + nn)},$$

poſito $\sqrt{(1 + nn)} = m$,

tota preſſio per elementum Yy erit

$$= m\,dx(\alpha + \beta x + \gamma y),$$

cuius ergo integrale eſt

$$m(\alpha x + \tfrac{1}{2}\beta x^2 + \gamma \int y\,dx) + C,$$

quod vt per totam datam rectam extendatur, primo talis conſtans adiici debet, vt poſito $x = AE$, euaneſcat, tum vero ſtatuatur $x = Ae$, quo facto ſumma preſſionum per Ff erit:

$$m\alpha Ee + \tfrac{1}{2}m\beta(Ae^2 - AE^2) + m\gamma.\,\text{Area}.\,EFef,$$

quae area quum ſit

$\tfrac{1}{2}Ee(EF + ef)$, et ob $Ae^2 - AE^2 = Ee(Ae + AE)$ fiet ſumma preſſionum per lineam Ff

$$= m\,Ee(\alpha + \tfrac{1}{2}\beta(AE + Ae) + \tfrac{1}{2}\gamma(EF + ef)).$$

 Quum

Quum porro preſſio per elementum Y y ſit

$$= m\,dx\,(\alpha + \beta x + \gamma y)$$

vt eius momentum prodeat reſpectu axis AB, ducatur ea in y, quod ergo erit

$$m\,y\,dx\,(\alpha + \beta x + \gamma y),$$

pro huius integratione iam vidimus per totam rectam Ff fore

$$\int y\,dx = \tfrac{1}{2}\,Ee\,(EF + ef),$$

bina reliqua integralia ob $y = e + nx$ ſeorſim euoluamus:

Pro littera β

$$\int y\,x\,dx = e\int x\,dx + n\int x^2\,dx$$

quod integrale per totam rectam Ff extenſum praebet

$$\tfrac{1}{2}e(Ae^2 - AE^2) + \tfrac{1}{3}n(Ae^3 - AE^3) = Ee\left(\tfrac{1}{2}e(Ae + AE) + \tfrac{1}{3}n(Ae^2 + Ae.AE + AE^2)\right)$$

Pro littera γ

$$\int y\,y\,dx = ee\int dx + 2en\int x\,dx + nn\int x^2\,dx,$$

quod integrale per totam rectam extenſum dat

$$ee.Ee + en.(Ae^2 - AE^2) + \tfrac{1}{3}n^2(Ae^3 - AE^3),$$

quae forma in hanc contrahitur:

$$Ee\left(ee + en(Ae + AE) + \tfrac{1}{3}nn(Ae^2 + Ae.AE + AE^2)\right)$$

Hinc ergo concludimus momentum reſpectu axis AB

$$mEe\left(\tfrac{1}{2}\alpha(EF + ef)\right) + mEe\left(\tfrac{1}{2}e\beta(Ae + AE) + \tfrac{1}{3}n\beta(Ae^2 + Ae.AE + AE^2)\right)$$
$$+ mEe\left(\gamma ee + \gamma en(Ae + AE) + \tfrac{1}{3}\gamma nn(Ae^2 + Ae.AE + AE^2)\right).$$

Calculus autem concinnior reddetur, ſi hinc litteras in subsidium vocatas e et n eliminemus, quum enim sumto

$x = \mathrm{AE}$, fiat $y = \mathrm{EF} = e + n.\mathrm{AE}$,

posito autem

$x = \mathrm{A}e$, fiat $y = ef = e + n\,\mathrm{A}e$,

subtrahendo elicimus

$n.\mathrm{E}e = ef - \mathrm{EF}$, $n = \frac{ef - \mathrm{EF}}{\mathrm{E}e}$, indeque $e = \frac{\mathrm{A}e.\mathrm{EF} - \mathrm{AE}.ef}{\mathrm{E}e}$,

at ex hoc valore n colligimus

$m = \sqrt{(1 + nn)} = \frac{\mathrm{F}f}{\mathrm{E}e}$;

ex quo valore summa ipsarum pressionum per rectam $\mathrm{F}f$ ita concinnius exprimitur:

$$\mathrm{F}f(\alpha + \tfrac{1}{2}\beta(\mathrm{A}e + \mathrm{AE}) + \tfrac{1}{2}\gamma(ef + \mathrm{EF})).$$

Tum vero pro momento respectu axis AB, singulae partes litteris α, β et γ affectae ita exprimentur:

Pro littera α habebimus

$$\tfrac{\alpha}{2}\mathrm{F}f(\mathrm{EF} + ef).$$

Pro littera β habebimus

$$\tfrac{\beta.\mathrm{F}f}{6}(ef(2\mathrm{A}e + \mathrm{AE}) + \mathrm{EF}(2\mathrm{AE} + \mathrm{A}e))$$

Pro littera γ fiet:

$$\tfrac{\gamma.\mathrm{F}f}{3}(ef^2 + ef.\mathrm{EF} + \mathrm{EF}^2).$$

Denique pro momento respectu alterius axis AC, novo calculo non est opus, sed sufficit in forma praecedenti, primo litteras β, γ, tum vero etiam rectas AE et EF, item Ae et ef inter se permutasse, sicque reperietur.

Momentum respectu axis A C

$$+\tfrac{\alpha}{2}\,Ff(AE + Ae)$$
$$+\tfrac{\beta}{3}\,Ff(Ae^2 + Ae.AE + AE^2)$$
$$+\tfrac{\gamma}{4}\,Ff(Ae(2ef + EF) + AE(2EF + e$$

Problema 4.

Tab. III. Fig. 10. Si pondus plano insistat; limbo triangulari A B D, definire pressionem in singulis punctis huius limbi.

Solutio.

23. Sit pressio totalis $= G$, cuius directio normaliter incidat in puncto O, vnde ad axem A ducatur perpendiculum OP, itemque ex angulo D perpendiculum D G. Quum iam limbus constet tribus lateribus trianguli A B, A D et B D, ad vnum quodque calculum Lemmatis praemissi seorsim accommodemus, ac quidem pro latere A B habebimus

$Ff = AB$; $AE = 0$; $Ae = AB$; $EF = 0$ et $ef = 0$

vnde colligitur

I°. pressio per $AB = AB(\alpha + \tfrac{1}{2}\beta.AB)$

II°. Momentum respectu axis $AB = 0$

III°. Momentum respectu axis $AC = AB(+\tfrac{\alpha}{2}AB + \tfrac{\beta}{3}AB$

Deinde pro latere A D

habebimus

$Ff = AD$, $AE = 0$, $Ae = AG$; $EF = 0$; $ef = DG$

vnde tria nostra momenta erunt:

1°. Ipsa pressio in latere $AD = AD(\alpha + \tfrac{\beta}{2}.AG + \tfrac{1}{2}\gamma DG$

2^{do} Momentum respectu axis $AB = AD(\frac{\alpha}{2}.DG + \frac{1}{3}DG.AG + \frac{\gamma}{3}DG^2)$

$$= AD \times DG(\frac{\alpha}{2} + \frac{1}{3}\beta\, AG + \frac{1}{3}\gamma\, DG)$$

3^{o} Momentum respectu axis $AC = AD(\frac{\alpha}{2}AG + \frac{1}{3}\beta\, AG^2 + \frac{1}{3}\gamma.AG.DG)$

$$= AD \times AG(\frac{\alpha}{2} + \frac{1}{3}\beta.AG + \frac{1}{3}\gamma.DG).$$

Pro tertio autem latere BD erit

$$Ff = BD,\ AE = AG;\ Ae = AB;\ EF = DG;\ ef = 0$$

vnde colligimus

1^{o}. Pressionem per latus $BD = BD(\alpha + \frac{1}{2}\beta(AB + AG) + \frac{1}{2}\gamma.DG)$

2^{do} Momentum respectu axis $AB = BD(\frac{\alpha}{2}DG + \frac{\beta}{6}(DG(2AG + AB) + \frac{\gamma}{3}DG^2)$

$$= BD \times DG(\frac{\alpha}{2} + \frac{\beta}{3}AG + \frac{1}{6}\beta AB + \frac{\gamma}{3}DG)$$

3^{o} Momentum respectu axis $AD = \frac{1}{2}\alpha BD(AG + AB)$

$$+ BD(\frac{\beta}{3}(AB^2 + AB.AG + AG^2) + \frac{1}{6}\gamma(AB.DG + 2AG.DG))$$

$$= BD \times AG(\frac{\alpha}{2} + \frac{\beta}{3}(\frac{1}{2}AB + AG) + \frac{1}{3}\gamma.DG)$$

$$+ BD.AB(\frac{\alpha}{2} + \frac{\beta}{3}(AB + \frac{1}{2}AG) + \frac{1}{6}.DG).$$

Quum igitur pressio totalis sit $= G$, eiusque momentum respectu axis $AB = G.OP$, at respectu axis $AC = G.AP$, consequimur tres sequentes aequationes:

$$\text{I. } G = \alpha(AB + BD + DC) + \frac{1}{2}\beta(AB^2 + AD \times AG + AB \times BD + AG \times BD) + \frac{1}{2}\gamma(DG.AD + DG \times BD)$$

$$= \alpha(AB + BD + DC) + \frac{1}{2}\beta(AB(AB + AD + BD) + AG.BD - BG.AD) + \frac{1}{2}\gamma DG(AD + BD)$$

$$\text{II. } G \times OP = \frac{\alpha}{2}(AD.DG + BD \times DG) + \frac{1}{3}\beta.DG(AG(AD + BD) + \frac{1}{2}BD.AB) + \frac{1}{3}\gamma DG^2(AD + BD).$$

$$\text{III. } G.\,AP = \tfrac{\alpha}{2}(AB^2 + AG.BD + AB.BD + AG \times AD)$$
$$+ \tfrac{\beta}{3}(AB^3 + AD.AG^2 + AB^2.BD + AB.AG.BD + AG^2.BD)$$
$$+ \tfrac{\gamma}{3}(AG.AD.DG + 2AB.DG.BD + AG.DG.BD).$$

Ex quibus ternas litteras α, β, γ determinare licebit, quibus inuentis, pressiones singulorum laterum totas cognoscemus, at pro quolibet puncto perimetri binis coordinatis x et y indicato, pressio vti assumsimus, erit $\alpha + \beta x + \gamma y$.

Coroll.

24. Si triangulum ABD fuerit aequilaterum, vnumque latus vocetur $= a$, erit $AG = \frac{1}{2} a$ et $DG = \frac{1}{2} a \sqrt{3}$, hocque casu ternae aequationes inventae, sequentes induent formas

$$\text{I. } G = 3\alpha a + \tfrac{3}{2}\beta a^2 + \tfrac{1}{2}\gamma . aa\sqrt{3}.$$
$$\text{II. } G.\,OP = \tfrac{\alpha}{2} aa\sqrt{3} + \tfrac{1}{4}\beta . a^3\sqrt{3} + \tfrac{1}{2}\gamma a^3.$$
$$\text{III. } G.\,AP = \tfrac{3}{2}\alpha . a^2 + \beta . a^3 + \tfrac{1}{2}\gamma a^3\sqrt{3},$$

hinc fit ex prima

$$\text{I}^{\circ}.\ \alpha a = \tfrac{1}{3} G - \tfrac{1}{2}\beta a^2 - \tfrac{1}{6}\gamma a^2\sqrt{3}$$

qui valor in binis reliquis substitutus praebet

$$\text{II}^{\circ}.\ \tfrac{G.\,OP}{a} = \tfrac{1}{2\sqrt{3}} G + \tfrac{1}{4}\gamma aa$$
$$\text{III}^{\circ}.\ \tfrac{G.\,AP}{a} = \tfrac{G}{2} + \tfrac{\beta}{4} aa + \tfrac{1}{4}\gamma aa\sqrt{3}.$$

Harum prior statim dat $\gamma aa = 4G\left(\frac{OP}{a} - \frac{1}{2\sqrt{3}}\right)$, tum vero ex postrema deducitur

$$\beta aa = \tfrac{4G}{a}(AP - OP\sqrt{3})$$

consequenter

$$\alpha a = G\left(\tfrac{2}{3} - \tfrac{2AP}{a} + \tfrac{4}{\sqrt{3}} . \tfrac{OP}{a}\right).$$

Ex

Ex his valoribus colligitur preſſio lateris

$AB = G\left(\frac{2}{3} - \frac{2}{\sqrt{3}} \cdot \frac{OP}{a}\right)$

preſſio lateris $AD = G\left(\frac{1}{3} - \frac{AP}{a} + \frac{4}{\sqrt{3}} \cdot \frac{OP}{a}\right)$

preſſio lateris $DB = G\left(\frac{1}{3} + \frac{AP}{a} - \frac{2}{\sqrt{3}} \cdot \frac{OP}{a}\right)$.

Problema 5.

Sit limbus quo pondus plano incumbit, parallelogrammum rectangulum ABCD, et directio totius preſſionis incidat in punctum O, inuenire preſſionem in ſingulis lateribus.

Solutio.

25. Vocemus latera $AB = CD = b$ et $AC = DB = c$, tum vero $AP = f$ et $PO = g$, exiſtente tota preſſione $= G$, nunc igitur ex Lemmate habebimus:

I°. Pro latere AB, $AE = 0$, $Ae = b$, $EF = 0$, $ef = 0$
et $Ff = AB = b$

ideoque,

I°. preſſionem ipſam $= AB(\alpha + \frac{1}{2}\beta . AB) = b(\alpha + \frac{1}{2}\beta b)$

II°. Moment. pro $AB = 0$

III°. Moment. reſpectu axis $AC = AB(\frac{\alpha}{2} . AB + \frac{\beta}{3} . AB^2)$
$= \frac{\alpha}{2} b^2 + \frac{1}{3}\beta . b^3$

II. Pro latere CD, $AE = 0$; $Ae = b$; $EF = ef = c$
et $Ff = AB = b$

I°.

I°. pressio ipsa $= AB(\alpha + \frac{1}{2}\beta . AB + \gamma . AC)$
$= \alpha b + \frac{1}{2}\beta . bb + \gamma bc$

II°. Moment. pro $AB = \alpha bc + \frac{1}{2}\beta bbc + \gamma bc^2$

III°. Moment. pro $AC = \frac{\alpha}{2}bb + \frac{1}{3}\beta b^3 + \frac{\gamma}{2}bbc$.

III. Pro latere AC, $AE = 0$; $Ae = 0$; $EF = 0$; $ef = c$;
et $Ff = c$

I. Ipsa pressio $= \alpha c + \frac{1}{2}\gamma cc$

II. Moment. respect. lateris $AB = \frac{1}{2}\alpha cc + \frac{1}{3}\gamma c^3$

III. Moment. respect. axis $AC = 0$,

IV. Pro latere BD; $AE = Ae = b$; $EF = 0$, $ef = c$, et $Ff = 0$.

I. Ipsa pressio $= \alpha c + \beta cb + \frac{1}{2}\gamma . cc$

II. Moment. resp. lateris $AB = \frac{\alpha}{2}c^2 + \frac{1}{2}\beta bcc + \frac{1}{3}\gamma c^3$

III. Moment. resp. lateris $AC = \alpha bc + \beta cbb + \frac{\gamma}{2}cbb$.

Atque hinc colligimus tres sequentes aequationes

I. $2\alpha(b+c) + \beta(bb+bc) + \gamma(bc+cc) = G$

II. $G.OP = \alpha(bc+cc) + \frac{1}{2}\beta\, cb(b+c) + \gamma c(bc + \frac{2}{3}c^2)$

III. $G.AP = \alpha(bb+bc) + \beta . bb(\frac{2}{3}b + c) + \frac{1}{2}\gamma . bc(b+c)$.

Ex quibus manifesto sequitur, posito $OP = g$ et $AP = f$

$\alpha(b+c) = \frac{G}{2} - \frac{\beta}{2}b(b+c) - \frac{\gamma c}{2}(b+c)$ ideoque

$$\gamma cc(\tfrac{1}{2}b + \tfrac{1}{6}c) = G(g - \tfrac{c}{2});\ \text{hinc}\ \gamma cc = \frac{G(2g-c)}{(b+\frac{1}{3}c)} = \frac{3G(2g-c)}{3b+c}$$

$$\beta bb(\tfrac{1}{2}c + \tfrac{1}{6}b) = G(f - \tfrac{b}{2})\ \text{et}\ \beta bb = \frac{G(2f-c)}{(c+\frac{1}{3}b)} = \frac{3G(2f-b)}{3c+b}.$$

Inuentis autem his valoribus α, β, γ, facile erit tam pro singulis lateribus, quam pro singulis eorum punctis, pressionem quam sustinent assignare.

Pro-

Problema 6.

Si limbus quo pondus plano incumbit, fuerit peripheria circuli centro A, radio $AB = Ab = a$ descripti, et directio pressionis totalis incidat in punctum O, pressiones in singulis peripheriae punctis assignare.

Solutio.

26. Diuidamus circulum in suos quatuor quadrantes, diametris BAb et CAc, qui simul vices nostrorum axium gerant, et consideremus primo solum quadrantem BAC, in quo sumamus punctum quodcunque Y, cuius vocemus abscissam $AX = x$ et applicatam $XY = y$, ita vt sit $y = \sqrt{(aa - xx)}$, et arcus CY elementum

$$= \frac{a\,dx}{\sqrt{(aa - xx)}} = \frac{a\,dx}{y},$$

iam per hoc elementum, pressio erit in Y

$$= \alpha + \beta x + \gamma y,$$

cum hoc autem puncto in reliquis quadrantibus simul coniungamus puncta analoga, Z, y et z, atque euidens est, pressionem fore in Z

$$= \alpha + \beta x - \gamma y;\ \text{in } y = \alpha - \beta x + \gamma y \text{ et in } z = \alpha - \beta x - \gamma y,$$

quarum pressionum summa est $= 4\alpha$, quae ducta in elementum arcus et per totum quadrantem CYB integrata, praebet nostram primam aequationem:

I. $G = 2\pi . aa$.

Nunc colligamus momenta respectu axis A B, quae ita se habebunt:

$$\begin{array}{lrl} \text{ex puncto} & Y = & \alpha y + \beta xy + \gamma \cdot yy \\ & Z = & -\alpha y - \beta xy + \gamma \cdot yy \\ & y = & +\alpha y - \beta xy + \gamma \cdot yy \\ & z = & -\alpha y + \beta xy + \gamma \cdot yy \\ \text{summa} & = & 4\gamma \cdot yy \end{array}$$

quae ducta in elementum arcus $\frac{a\,dx}{y}$, dat formulam integrandam $4\gamma\, a y\, dx$, at pro toto quadrante fit $\int y\, dx = \frac{1}{4}\pi\, a a$, vnde quum momentum totius pressionis fit $G.\,OP = G.\,g$ (posito $OP = g$) habebimus hanc secundam aequationem $G.\,g = \pi\gamma a^3$.
Denique pro axe A C, momenta nascuntur

$$\begin{array}{lrl} \text{ex momento} & Y = & \alpha x + \beta xx + \gamma xy \\ & Z = & \alpha x + \beta xx - \gamma xy \\ & y = & -\alpha x + \beta xx - \gamma xy \\ & z = & -\alpha x + \beta xx - \gamma xy \\ \text{summa} & = & +4\beta xx, \end{array}$$

quae in elementum $\frac{a\,dx}{\sqrt{(aa - xx)}}$ ducta, dat formulam integrandam

$\frac{4\beta a x^2\, dx}{\sqrt{(aa - xx)}}$, at $\int \frac{x^2\, dx}{\sqrt{(a^2 - xx)}} = a \int \frac{a\, dx}{\sqrt{(aa - xx)}} - \int dx \sqrt{(aa - xx)}$

Iam vero pro totum quadrantem fit

$$\int \frac{a\, dx}{\sqrt{(aa - xx)}} = \frac{\pi}{2} a, \text{ et } \int dx \sqrt{(aa - xx)} = \int y\, dx = \tfrac{1}{4}\pi a a$$

vnde colligitur

$$\int \frac{x^2\, dx}{\sqrt{(aa - xx)}} = \frac{\pi}{4} a a,$$

vnde quum totum momentum fit $G.\,AP = G.\,f$ posito $AP = f$, tertia aequatio nostra prodibit $G.\,f = \pi.\,\beta.\,a^4$.

Ergo

Ergo hinc ſtatim deducimus

$$\beta = \frac{G.f}{\pi.a^3};\quad \gamma = \frac{G.g}{\pi.a^3}\quad \text{et}\quad \alpha = \frac{G}{2\pi a},$$

quocirca pro puncto peripheriae quocunque Y, preſſio erit

$$\frac{G}{2\pi a} + \frac{Gfx}{\pi.a^3} + \frac{Ggy}{\pi.a^3} = \frac{G}{\pi a}\left(\frac{1}{2} + \frac{fx+gy}{aa}\right),$$

vnde pro ſingulis punctis preſſio eſt manifeſta.

Scholion.

27. Hactenus ponderi plano incumbenti eiusmodi dedimus baſin, quae vel tantum ex aliquot conſtaret punctis, vel in limbum linearem deſineret, nunc igitur eiusmodi baſes aggrediamur, quae ſecundum binas dimenſiones ſint extenſae, ac primo quidem pro huiusmodi caſibus, quibus baſis eſt vel triangulum, vel alia figura rectilinea, ſequens Lemma praemittamus.

Lemma.

Si trapezium $EFef$ fuerit portio baſis cuiuscunque, qua pondus plano incumbit, definire tam totam preſſionem, quam hoc trapezium ſuſtinet, quam eius momenta reſpectu binorum axium inter ſe normalium AB et AC.

Solutio.

28. Quum rectae FE et fe ſumantur ad axem AB normales, conſideretur quaecunque intermedia YX illis parallela, itemque huic proxima xy, et conſideretur elementum quodcunque $VvUu$ Tab. III. Fig. 13.

 rectan-

rectangulum, et pro puncto V statuantur coordinatae A X $= x$ et X V $= v$, eritque pressio in puncto V $= \alpha + \beta x + \gamma v$, quae quia per totum rectangulum $V v U u$, cuius area est $dx\,dv$, valere concipitur, erit pressio quam hoc elementum sustinet

$$= \alpha\, dx\, dv + \beta\, x\, dx\, dv + \gamma\, v\, dx\, dv,$$

tum vero eius momentum respectu axis A B

$$= \alpha\, v\, dx\, dv + \beta\, x\, v\, dx\, dv + \gamma\, v\, v\, dx\, dv$$

et respectu axis A C momentum

$$= \alpha\, x\, dx\, dv + \beta\, x^2\, dx\, dv + \gamma\, v\, x\, dx\, dv$$

quas singulas partes, duplici integratione tractari oportet, primo igitur abscissam x vt constantem spectemus, et integralia per totam fasciolam elementarem X Y $x y$ extendamus, quod fit faciendo post integrationem $v =$ X Y $= y$, hocque modo colligemus:

Pro fasciola X Y $x y$

I. Pressionem $= \alpha y\, dx + \beta y x\, dx + \frac{1}{2}\gamma y y\, dx$

II. Momentum respectu axis AB $= \frac{1}{2}\alpha y y\, dx + \frac{1}{2}\beta y y x\, dx + \frac{1}{3}\gamma y^3\, dx$

III. Momentum respectu axis AC $= \alpha y x\, dx + \beta y x^2\, dx + \frac{1}{2}\gamma y^2 x\, dx$

tantum igitur superest, vt singulas has formulas altera vice integremus, et per totam aream trapezii extendamus quod fiet, si integralia euanescentia reddantur, ponendo $x =$ A E et $y =$ E F, tum vero statuatur $x =$ A e et $y = e f$, in hunc finem, quia linea F f est recta, statuatur $dy = n\, dx$ eritque $n = \frac{ef - \mathrm{EF}}{\mathrm{E}e}$, integralia autem ita per notam reductionem expediamus, secundum formulam.

$\int p\,dq = pq - \int q\,dp$.

Hoc praenotato erit:

1°. $\int y\,dx = yx - \frac{1}{2}nx^2$, ergo pro toto trapezio

$$\int y\,dx = \mathrm{A}e.\,ef - \mathrm{AE.\,EF} - \tfrac{1}{2}n(\mathrm{A}e^2 - \mathrm{AE}^2)$$

quam formulam quo facilius euoluamus, statuamus

$$\mathrm{AE} = \mathrm{E};\ \mathrm{EF} = \mathrm{F};\ \mathrm{A}e = e \text{ et } ef = f,$$

ita vt sit $n = \frac{f - \mathrm{F}}{e - \mathrm{E}}$. Ideoque

$$\int y\,dx = \tfrac{1}{2}(e - \mathrm{E})(f + \mathrm{F})$$

2°. $\int yx\,dx = \frac{1}{2}yx^2 - \frac{n}{2}\int x^2\,dx = \frac{1}{2}yx^2 - \frac{n}{6}x^3$, ergo

$$\int yx\,dx = \tfrac{1}{2}eef - \tfrac{1}{2}\mathrm{EEF} - \frac{(f-\mathrm{F})}{6(e-\mathrm{E})}(e^3 - \mathrm{E}^3)$$

$$= \tfrac{1}{2}eef - \tfrac{1}{2}\mathrm{EEF} - \frac{(f-\mathrm{F})}{6}(ee + e\mathrm{E} + \mathrm{E}^2), \text{ ergo}$$

$$\int yx\,dx = \tfrac{1}{6}(e - \mathrm{E})(f(2e + \mathrm{E}) + \mathrm{F}(2\mathrm{E} + e))$$

3°. $\int yy\,dx = \frac{1}{n}\int y^2\,dy = \frac{1}{3n}y^3$. Ideoque

$$\int yy\,dx = \tfrac{1}{3}(e - \mathrm{E})(ff + f\mathrm{F} + \mathrm{F}^2)$$

4°. $\int yyx\,dx = \frac{xy^3}{3n} - \frac{y^4}{12n^2}$. Ideoque

$$\int yyx\,dx = \tfrac{1}{12}(e-\mathrm{E})(e(3f^2 + 2f\mathrm{F} + \mathrm{F}^2) + \mathrm{E}(3\mathrm{F}^2 + 2\mathrm{F}f + f^2))$$

siue $\frac{1}{12}(e - \mathrm{E})(2ef^2 + 2\mathrm{EF}^2 + (e + \mathrm{E})(f + \mathrm{F})^2)$

5°. $\int y^3\,dx = \frac{1}{n}\int y^3\,dy = \frac{1}{4n}y^4$. Ideoque

$$\int y^3\,dx = \tfrac{1}{4}(e - \mathrm{E})(f^3 + ff\mathrm{F} + f\mathrm{F}^2 + \mathrm{F}^3)$$

6°. $\int yx^2\,dx = \frac{1}{3}yx^3 - \frac{1}{3}\int x^3\,dy = \frac{1}{3}yx^3 - \frac{n}{12}x^4$. Ideoque

$$\int yx^2\,dx = \tfrac{1}{12}(e-\mathrm{E})(f(3e^2 + 2e\mathrm{E} + \mathrm{E}^2) + \mathrm{F}(3\mathrm{E}^2 + 2\mathrm{E}e + e^2))$$

siue $\frac{1}{12}(e - \mathrm{E})(2fe^2 + 2\mathrm{FE}^2 + (f + \mathrm{F})(e + \mathrm{E})^2)$.

Quocirca tres formulae principales quas inuenimus sequenti modo exprimentur:

I. Pressio $=\frac{1}{2}\alpha(e-E)(f+F)+\frac{1}{6}\beta(e-E)(f(2e+E)+F(2E+e)) + \frac{1}{6}\gamma(e-E)(ff+fF+F^2)$

II. Momentum respectu $AB=\frac{1}{6}\alpha(e-E)(ff+fF+F^2) + \frac{1}{24}\beta(e-E)(2ef^2+2EF^2+(e+E)(f+F)^2) + \frac{1}{12}\gamma(e-E)(f^3+ffF+fF^2+F^3)$

III. Moment. respectu axis $AC=\frac{1}{6}\alpha(e-E)(f(2e+E)+F(2E+e)) + \frac{1}{12}\beta(e-E)(2fe^2+2FE^2+(f+F)(e+E)^2) + \frac{1}{24}\gamma(e-E)(2ef^2+2EF^2+(e+E)(f+F)^2)$

Problema 7.

Fig. 10. Si basis qua pondus plano incumbit, fuerit triangulum ABD et media directio pressionis totalis G cadat in punctum O, inuenire pressionem in singulis baseos punctis.

Solutio.

29. In basin AB demisso perpendiculo DG vocentur $AG=a$ et $BG=b$ et $DG=c$, tum vero sit $AP=p$ et $PO=q$ et quia basis constat partibus AGD et GDB, ad vtramque Lemma praecedens accommodemus, ac primo pro spatio ADG, habebimus $E=0$, $F=0$, $e=a$, $f=c$, hincque tres formulae nostrae erunt

I. Pressio $=\frac{1}{2}\alpha.ac+\frac{1}{3}\beta a^2c+\frac{1}{6}\gamma acc$

II. Moment. respect. $AB=\frac{1}{6}\alpha acc+\frac{1}{8}\beta aacc+\frac{1}{12}\gamma ac^3$

III. Moment. respect. $AC=\frac{1}{6}\alpha aac+\frac{1}{4}\beta a^3c+\frac{1}{8}\gamma a^2c^2$

Pro

Pro altero autem triangulo G D B, quia $E = a$, $F = c$, $e = a + b$ et $f = 0$, habebimus

I. Pressionem $= \frac{1}{2}\alpha . bc + \frac{1}{6}\beta . bc(3a+b) + \frac{1}{6}\gamma bcc$

II. Moment. resp. AB $= \frac{1}{6}\alpha bcc + \frac{1}{24}\beta . bcc(4a+b) + \frac{1}{12}\gamma bc^3$

III. Moment. resp. AC $= \frac{1}{6}\alpha bc(3a+b) + \frac{1}{12}\beta bc(6aa+4ab+bb)$
$+ \frac{1}{24}\gamma bcc(4a+b)$.

His igitur coniungendis nanciscimur tres sequentes aequationes:

I. $G = \frac{1}{2}\alpha c(a+b) + \frac{1}{6}\beta c(2a^2+3ab+bb) + \frac{1}{6}\gamma cc(a+b)$
$= (a+b)(\frac{1}{2}\alpha c + \frac{1}{6}\beta c(2a+b) + \frac{1}{6}\gamma cc)$

II. $Gq = \frac{1}{6}\alpha cc(a+b) + \frac{1}{24}\beta cc(3aa+4ab+bb) + \frac{1}{12}\gamma c^3(a+b)$

III. $Gp = \frac{1}{6}\alpha c(2a^2+3ab+bb) + \frac{1}{12}\beta c(3a^3+6aab+4abb+b^3)$
$+ \frac{1}{24}\gamma cc(3aa+4ab+bb)$
$= \frac{1}{6}\alpha c(a+b)(2a+b) + \frac{1}{12}\beta c^2(a+b)(3aa+3ab+bb)$
$+ \frac{1}{24}\gamma cc(a+b)(3a+b)$.

Quarum secunda in $\frac{3}{c}$ ducta subtrahatur a prima, et remanebit

$$G - \frac{3Gq}{c} = G\left(1 - \frac{3q}{c}\right) = -\frac{\beta c}{24}(a+b)(a-b) - \frac{1}{12}\gamma cc(a+b)$$

Deinde tertia ducta in 3 a prima in $2a+b$ ducta subtrahatur et prodibit

$$G(2a+b-3p) = -\frac{1}{12}\beta c(a+b)(aa+ab+bb) - \frac{1}{24}\gamma cc(a+b)(a-b).$$

Quarum prior ducta in $(a-b)$ si a duplo posterioris subtrahatur relinquet:

$$G\left(4a+2b-6p-(a-b)\left(1-\frac{3q}{c}\right)\right) = -\frac{1}{8}\beta c(a+b)^3 \text{ siue}$$
$$G\left(a+b-2p+(a-b)\frac{q}{c}\right) = -\frac{1}{24}\beta c(a+b)^3$$

vnde

vnde β determinatur ex quo deinceps et γ et α innotescent.

Est vero

$$-\tfrac{1}{24}\gamma cc(a+b)^3 = G(b(a+b)+p(a-b)-2(aa+ab+bb))$$

et $+\tfrac{1}{6}\alpha c(a+b)^2 = G(3(a+b)-4p-\tfrac{4bq}{c})$.

Corollarium I.

30. Si basis fuerit triangulum rectangulum AGD, quod fit si $b=0$ ternae nostrae aequationes erunt:

$$\text{I. } G = \tfrac{1}{2}\alpha ac + \tfrac{1}{3}\beta a^2c + \tfrac{1}{6}\gamma acc$$

$$\text{II. } \tfrac{Gq}{c} = \tfrac{1}{6}\alpha ac + \tfrac{1}{8}\beta a^2c + \tfrac{1}{12}\gamma acc$$

$$\text{III. } \tfrac{Gp}{a} = \tfrac{1}{3}\alpha ac + \tfrac{1}{4}\beta a^2c + \tfrac{1}{8}\gamma acc.$$

Hinc

$$\tfrac{1}{18}\alpha ac = G(3-\tfrac{4p}{a})$$

$$\tfrac{1}{24}\beta aac = G(\tfrac{2p}{a}-\tfrac{q}{c}-1)$$

$$\tfrac{1}{24}\gamma acc = G(\tfrac{2q}{c}-\tfrac{p}{a})$$

atque hinc pro quouis puncto huius basis, binis coordinatis x et y determinato, pressio erit

$$=\alpha+\beta x+\gamma y.$$

Coroll. 2.

31. Si basis fuerit triangulum Isosceles quod euenit si $b=a$, ternae aequationes nostrae sunt

$$G = \alpha ac + \beta aac + \tfrac{1}{3}\gamma acc$$

$$\tfrac{Gq}{c} = \tfrac{1}{3}\alpha ac + \tfrac{1}{3}\beta aac + \tfrac{1}{6}\gamma acc$$

$$\tfrac{Gp}{a} = \alpha ac + \tfrac{7}{6}\beta aac + \tfrac{1}{3}\gamma acc$$

Hinc

Hinc fit

$$\tfrac{1}{2}\beta aac = G\left(\tfrac{p}{a}-1\right);\ \tfrac{1}{6}\gamma acc = G\left(\tfrac{3q}{c}-1\right);\ \tfrac{1}{3}\alpha ac = G\left(3-\tfrac{2q}{c}-\tfrac{2p}{a}\right).$$

Si praeterea fuerit $p=a$, ita vt punctum O cadat in perpendiculum D G, erit

$$\beta = 0;\ \tfrac{1}{3}\alpha ac = G\left(1-\tfrac{2q}{c}\right)\ \text{fiue}$$

$$\alpha = \tfrac{3G}{ac}\left(1-\tfrac{2q}{c}\right);\ \beta = 0;\ \gamma = \tfrac{6G}{acc}\left(\tfrac{3q}{c}-1\right).$$

Si fuerit $q=c$ erit

$$\alpha = -\tfrac{3G}{ac};\quad \beta = 0;\quad \gamma = +\tfrac{12G}{acc}.$$

Sin autem fuerit $q=\frac{1}{3}c$ quo cafu punctum O in ipfum grauitatis trianguli cadit, fiet etiam $\gamma=0$, $\alpha=\frac{G}{ac}$, et preffio vbique erit conftans.

Coroll. 3.

32. Quodfi vero in formulis generalibus ftatim ponamus $\beta=0$ et $\gamma=0$; quia tum eft $\alpha=\frac{2G}{c(a+b)}$, reperiemus $p=\frac{2a+b}{3}$ et $q=\frac{1}{3}c$, ficque punctum O incidet in ipfum centrum grauitatis trianguli.

Problema.

Si bafis qua pondus plano incumbit, fuerit parallelogrammum rectangulum A B C D et directio preffionis totalis incidat in punctum O, affignare preffionem in fingulis punctis. Fig. 10.

Solutio.

33. Ponamus vt fupra $AB=b$ et $AC=c$, item $AP=p$ et $PO=q$ vnde pro noftro Lem-

 mate

mate erit $E = 0$; $e = b$; $F = c$ et $f = c$, hincque ſtatim obtinentur ſequentes tres aequationes

$$\text{I. } G = \alpha bc + \tfrac{1}{2}\beta bbc + \tfrac{1}{2}\gamma bcc$$

$$\text{II. } \tfrac{Gq}{c} = \tfrac{1}{2}\alpha bc + \tfrac{1}{4}\beta bbc + \tfrac{1}{3}\gamma bcc$$

$$\text{III. } \tfrac{Gp}{b} = \tfrac{1}{2}\alpha bc + \tfrac{1}{3}\beta bbc + \tfrac{1}{4}\gamma bcc$$

vnde ſtatim concludimus, a tertia bis ſumta ſubtrahendo primam

$$G\left(\tfrac{2p}{b} - 1\right) = \tfrac{1}{6}\beta bbc; \text{ ergo } \beta bbc = 6G\left(\tfrac{2p}{b} - 1\right).$$

At ſi a ſecunda bis ſumta ſubtrahatur prima, reperitur

$$G\left(1 - \tfrac{2q}{c}\right) = \tfrac{1}{6}\gamma bcc \text{ ideoque } \gamma bcc = 6G\left(\tfrac{2q}{c} - 1\right)$$

hincque

$$\alpha bc = G\left(7 - \tfrac{6p}{b} - \tfrac{6q}{c}\right).$$

Nunc pro puncto quocunque coordinatis x et y definito, preſſio erit $\alpha + \beta x + \gamma y$.

Coroll. I.

34. Hinc in puncto A vbi $y = 0$ et $x = 0$ preſſio erit

$$= \tfrac{G}{bc}\left(7 - \tfrac{6p}{b} - \tfrac{6q}{c}\right).$$

In angulo vero B preſſio prodit

$$\alpha + \beta b = \tfrac{G}{bc}\left(1 + \tfrac{6p}{b} - \tfrac{6q}{c}\right),$$

porro preſſio in C erit $= \alpha + \gamma c = \tfrac{G}{bc}\left(1 - \tfrac{6p}{b} + \tfrac{6q}{c}\right)$

denique preſſio in puncto D erit $= \alpha + \beta b + \gamma c = \tfrac{G}{bc}\left(\tfrac{6p}{b} + \tfrac{6q}{c} - 5\right)$

Coroll.

Coroll. 2.

35. Si fuerit $p=\frac{1}{2}b$ et $q=\frac{1}{2}c$, quo casu punctum O in medium rectanguli incidit, fiet

$$\alpha=\frac{G}{bc};\quad \beta=0;\quad \gamma=0$$

vnde hoc casu per totam basin pressio aequaliter distribuetur.

Problema.

Si basis, qua corpus plano incumbit fuerit circulus radio $AB=a$ descriptus, et directio pressionis totius G, cadat in punctum O, vt sit $AP=p$ et $PO=q$, inuenire pressionem in singulis punctis. Tab. III. Fig. 12.

Solutio.

36. Diuiso vt supra circulo in suos quatuor quadrantes, in primo consideretur punctum quodcunque V, pro quo ponatur $AX=x$ et $XV=v$, ibique erit pressio $=\alpha+\beta x+\gamma v$, simul vero considerentur in reliquis quadrantibus, puncta analoga v, U, u, ac primo quidem pressiones in his punctis ita se habebunt:

$$\begin{aligned}
\text{I. Pressio in } V &= \alpha+\beta x+\gamma v\\
\text{in } v &= \alpha-\beta x+\gamma v\\
\text{in } U &= \alpha+\beta x-\gamma v\\
\text{in } u &= \alpha-\beta x-\gamma v\\
\text{Summa} &= 4\alpha.
\end{aligned}$$

 II.

II. Secundo momenta respectu axis A B erunt sequentia

pro puncto $V = \alpha v + \beta x v + \gamma v v$

$v = \alpha v - \beta x v + \gamma v v$

$U = - \alpha v - \beta x v + \gamma v v$

$u = - \alpha v + \beta x v + \gamma v v.$

Summa $= + 4 \gamma v v$

III. Momenta respectu axis A C erunt

pro puncto $V = \alpha x + \beta x x + \gamma x v$

$v = - \alpha x + \beta x x - \gamma x v$

$U = + \alpha x + \beta x x - \gamma x v$

$u = - \alpha x + \beta x x + \gamma x v$

Summa $= + 4 \beta x x.$

Hae formulae ducantur in elementum areae quod est $d x d v$ ac primo sumatur x constans et facta integratione ponatur

$$v = XY = y = \sqrt{(a a - x x)}$$

et habebimus

$$4 \alpha \int d x d v = 4 \alpha d x \sqrt{(a a - x x)}$$

$$4 \gamma \int v v d v d x = \tfrac{4}{3} \gamma y^3 d x = \tfrac{4}{3} \gamma d x (a a - x x)^{\frac{3}{2}}$$

$$4 \beta \int x x d x d v = 4 \beta y x^2 d x = 4 \beta x^2 d x (a a - x x)^{\frac{1}{2}}$$

Hae formulae denuo integrentur per totum quadrantem B A C, ac reperietur

$$\int y d x = \tfrac{1}{4} \pi a a; \text{ hincque } 4 \alpha \int y d x = \alpha \pi a a$$

$$\int y^3 d x = \tfrac{1}{4} x (a a - x x)^{3:2} + \tfrac{3}{4} a^2 \int d x \sqrt{(a a - x x)},$$

per totum quadrantem

$$\int y^3 d x = \tfrac{3}{16} \pi . a^4, \text{ hinc } \tfrac{4}{3} \gamma \int y^3 d x = \tfrac{1}{4} \gamma \pi a^4$$

deni-

denique

$$\int yx^2dx = \tfrac{1}{16}\pi a^4 \text{ et } 4\beta\int yx^2dx = \tfrac{1}{4}\beta\pi a^4,$$

atque hinc tres noſtrae aequationes erunt:

$$\text{I. } G = \alpha\pi aa;\ \text{II. } Gq = \tfrac{1}{4}\gamma\pi a^4;\ \text{III. } Gp = \tfrac{1}{4}\beta\pi a^4.$$

Ideoque

$$\alpha = \frac{G}{\pi aa};\quad \beta = \frac{4Gp}{\pi a^4};\quad \gamma = \frac{4Gq}{\pi a^4}.$$

Coroll. 1.

37. Si punctum V incidat in ipſum centrum circuli ſeu punctum A, habebitur preſſio in iſto puncto $= \frac{G}{\pi aa}$, euaneſcentibus ſcilicet x et v. Hinc vero patet punctum A eandem ſuſtinere preſſionem, vbicunque demum inciderit punctum O.

Coroll. 2.

38. Si punctum V cadat in ipſum punctum O, hincque fuerit $x = p$, $y = q$, habebitur preſſio in puncto

$$O = \frac{G}{\pi aa}\left(1 + \frac{4(pp+qq)}{aa}\right) = \frac{G}{\pi aa}\left(1 + \frac{4AO^2}{AB^2}\right)$$

Scholion.

39. Tot caſibus ſpecialibus hactenus euolutis, quibus hoc nouum argumentum haud mediocriter illuſtratur, nunc demum conueniet, ſolutionem maxime generalem tradere, quae principiis in Mechanicae meae Tractato Tertio expoſitis innititur.

Problema Generale.

Quamcunque figuram habuerit basis, qua pondus plano incumbit, determinare pressionem in singulis baseos elementis.

Solutio.

Tab. III. Fig. 14. 40. Repraesentet figura $EFef$ basin propositam, cuius centrum grauitatis sit in puncto G, per quod ducti sint bini axes principales eiusdem figurae EGe et FGf, quemadmodum in Mechanicae loco citato sunt constituti. Deinde secundum principia ibidem stabilita, quaerantur momenta inertiae respectu eorundem axium, quae scilicet reperiuntur, si singula baseos elementa in quadrata distantiarum ab iisdem axibus multiplicentur. Denotet igitur A aream totius huius basis $EFef$ et respectu axis Ee sit momentum inertiae $= A.ee$ respectu autem alterius axis sit $= Aff$, tum vero media directio pressionis totalis incidat in punctum O, cuius distantiae ab axibus principalibus sint $OP = p$ et $OQ = q$. His praemissis, consideremus areae quodcunque elementum in y, pro quo vocentur coordinatae $GX = x$ et $XY = y$, ex quibus ipsum areae elementum fit $dx.dy$. Iam quia G est centrum grauitatis totius figurae, bina haec integralia duplicata $\iint x\,dx\,dy$ et $\iint y\,dx\,dy$ per totam figuram extensa euanescunt. Deinde natura axium principalium in hoc consistit, vt haec formula $\iint xy\,dx\,dy$ per totam figuram sumta etiam euanescat. Porro autem

autem formulae integrales per totam figuram extensae praebent:

I°. $\iint dx\,dy = A$; II°. $\iint yy\,dx\,dy = A\,ee$ et tertio $\iint xx\,dy\,dx = A\,ff$.

Quodsi ergo secundum principia supra stabilita, pressionem in puncto Y ponamus

$$= \alpha + \beta x + \gamma y,$$

ita vt pressio in ipsum elementum $dx\,dy$ sit

$$= \alpha\,dx\,dy + \beta x\,dx\,dy + \gamma y\,dx\,dy,$$

eius integrale ipsi pressioni totali, quae sit $= \Pi$, aequari debet, vnde quum sit

$$\iint dx\,dy = A;\ \iint x\,dx\,dy = 0 \text{ et } \iint y\,dx\,dy = 0,$$

assequimur hanc aequationem primam $\Pi = \alpha A$.

Momentum autem pressionis istius elementaris respectu axis E e, quod est

$$= \alpha y\,dx\,dy + \beta xy\,dx\,dy + \gamma yy\,dx\,dy,$$

per duplicem integrationem dare debet momentum totius $= \Pi.\,OP = \Pi p$, vnde deducitur haec aequatio $\Pi.\,p = \gamma A.\,ee$. Eodem denique modo pro altero axe Ff colligitur tertia nostra aequatio, $\Pi q = \beta A.\,ff$, atque hinc sponte prodeunt sequentes valores

$$\alpha = \frac{\Pi}{A};\quad \beta = \frac{\Pi q}{A.\,ff};\quad \gamma = \frac{\Pi p}{A.\,ee},$$

ita vt iam pro puncto y pressio sit

$$= \frac{\Pi}{A}\left(1 + \frac{qx}{ff} + \frac{py}{ee}\right).$$

Quo indolem et varietatem harum pressionum, pro diuersis locis clarius perspiciamus, quaeramus

ramus

ramus primo omnia loca, vbi pressio plane eva-nescit, quae quum in hac aequatione generali $1 + \frac{qx}{ff} + \frac{py}{ee} = 0$ contineantur, euidens est, ea in linea recta esse dispofita, ad quam inuestigandam faciamus primo $x = 0$, eritque $y = -\frac{ee}{p}$, sumto autem $y = 0$, fit $x = -\frac{ff}{q}$. Quocirca si capiamus

Tab. III. Fig. 15. $GM = \frac{ee}{p}$ et $GN = \frac{ff}{q}$, in punctis M et N pressio erit nulla, ideoque etiam per totam rectam MN. Plerumque haec recta MN extra ipsam basin corporis cadit, sin autem per ipsam basin transiret, tum casus supra memoratus locum esset habiturus, vbi in quapiam basis portione pressio negatiua esset inuenta, quod cum in praxi euenire nequeat, nostra solutio etiam ad praxin accommodari non potest. Sumamus igitur totam rectam MN extra basin propositam cadere, atque euidens est, si in ipsa basi, vbicunque ducatur recta huic MN parallela mn, per totam hanc chordam mn pressionem vbique eandem fore, atque si per ipsum punctum G huiusmodi parallela agatur, per eam vbique similis pressio regnabit, tanta scilicet quanta est in ipso G, vbi $x = 0$ et $y = 0$, ita vt per hanc rectam pressio futura sit $= \frac{\Pi}{A}$. Egregie haec conueniunt cum iis quae initio, circa principium nostrum generale sunt proposita, haec enim recta MN vbi pressiones euanescunt, est ipsa illa intersectio (Fig. 3. FG), vbi planum omnes pressiones repraesentans, planum Tabulae intersecat, atque hinc manifestum est, per omnes rectas huic MN parallelas, pressiones eo

fore

fore maiores quo magis fuerint a recta M N remotae. Sic quum in M pressio esset nulla, in G vero $\frac{\Pi}{A}$, in alio puncto μ erit $\frac{\Pi . M\mu}{A . MG}$, vnde manifesto sequitur in eo basis puncto, pressionem omnium fore maximam, quod a recta M N maxime fuerit remotum, quandoquidem pressiones per totam basin in ratione distantiarum a recta M N increscunt.

Coroll. 1.

41. Si punctum O incidat in ipsum centrum grauitatis basis, vt sit $p = o$ et $q = o$, tum puncta M et N in infinitum elongabuntur, ideoque per totam basin aequabiliter distribuetur, in singulis quippe punctis erit $= \frac{\Pi}{A}$ ob $\beta = o$ et $\gamma = o$.

Coroll. 2.

42. Quodsi punctum O in alterum axem principalem cadat veluti in P, vt sit $OP = p = o$ et $GP = q$, tum punctum M in infinitum distabit at $GN = \frac{ff}{q}$ ipsa ergo recta M N alteri axi principali Ff erit parallela, vbi scilicet pressio euanescit, et quia pressio per totum istum axem Ee est $= \frac{\Pi}{A}$, hinc pressio per omnes rectas huic axi parallelas facile definitur.